

國立臺北科技大學 113 學年度碩士班招生考試  
系所組別：2131 電機工程系碩士班丙組

第一節 工程數學 試題（選考）

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注意事項：

1. 本試題共六題，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (10%) Solve the initial value problem  $4\sin^2(y)dx - \sec^2(x)dy = 0$ ;  $y(\frac{\pi}{4}) = 1$ .

2. Consider the differential equation  $y'' - 4y' + 3y = x + e^x + xe^x + e^{3x}$ .

- (a) (5%) Find the associate homogeneous solution.
- (b) (15%) Find the general solution.

3. (20%) Solve the differential equation:

$y''(t) + 6y'(t) + 10y(t) = f(t); y(0) = y'(0) = 0$  where

$$f(t) = \begin{cases} 0, & t < 2 \\ +3, & 2 \leq t < 3 \\ -3, & t \geq 3 \end{cases}$$

4. (15%) Find the inverse Laplace transform of  $(sI - A)^{-1}$  where

$$A = \begin{bmatrix} -4 & 1 & 0 \\ 0 & -4 & -6 \\ 0 & 0 & -4 \end{bmatrix}.$$

5. Consider the matrix  $A = \frac{\sqrt{2}}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & 0 \end{bmatrix}$ .

- (a) (5%) Find the eigenvalues of  $A^T A$ .
- (b) (10%) Find the expression of all eigenvectors of  $AA^T$ .

6. Consider the set  $S = \{(1, 0, 1), (2, 1, 0), (2, 2, 1)\}$ .

- (a) (10%) Derive the orthonormal set  $\hat{S}$  from  $S$ .
- (b) (10%) Express  $\mathbf{x} = (2\sqrt{2}, 0, \sqrt{2})$  as a linear combination of the vectors in the orthonormal set  $\hat{S}$ .