國立臺北科技大學 113 學年度碩士班招生考試

系所組別：1501、1502 自動化科技研究所

第一節　工程數學　試題

注意事項：

1. 本試題共 6 題，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Solve the following differential equation: (16%)

$$y'' - y' - 2y = 2x^2 + 5$$

2. Determine which of the following matrices are invertible, give your explanation.

(a) $\begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & 1 \\ -2 & 2 & -1 \end{bmatrix}$ (6%)

(b) $\begin{bmatrix} 1 & 3 & 0 \\ 2 & 1 & 0 \\ -2 & 2 & 0 \end{bmatrix}$ (6%)

3. Let S be the plane with the equation $x + 3y - z = 0$.
   (a) Find a basis for S. (6%)
   (b) Find the projection of the point a = (0, 1, 2) onto the plane S. (6%)

4. Solve the following differential equation: (16%)

$$x^2 y'' - 2xy' + 2y = \ln(x) + 1.$$

5. Solve the system (20%)

$$y_1' = 3y_1 + 4y_2$$
$$y_2' = 3y_1 + 2y_2$$

   with the initial value $y_1(0) = 6$, $y_2(0) = 1$.

6. Apply the Laplace transform to solve the initial problem (24%)

$$y'' + 4y' + 3y = e^t; \quad y(0) = 0, \quad y'(0) = 2.$$