112EE03

國立臺北科技大學 112 學年度碩士班招生考試

系所組別：2131 電機工程系碩士班丙組

第一節　工程數學　試題（選考）

注意事項：

1. 本試題共六題，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Consider the differential equation $4xy''(x)+x^2y'(x)+2xy(x)=[4xy'(x)]'+[F(x)y(x)]'=0$.

(a) Determine $F(x)$. (5 分)

(b) Find the general solution of the differential equation. (10 分)

2. Consider the differential equation $y'(x)+y(x)=y^4(x)$.

(a) Find the integrating factor of the differential equation. (10 分)

(b) Find the general solution of the differential equation. (5 分)

3. Consider the boundary value problem $t(1-t)y''(t)+2y'(t)+2y(t)=12t$;
$y(0)=0$, $y(3)=10$.

(a) Find $Y(s)$, in which $Y(s)$ is the Laplace transform of $y(t)$. (10 分)

(b) Solve the boundary value problem when $Y(s)$ is satisfied $\lim_{s\to\infty}Y(s)=0$. (10 分)

4. Consider the quadratic form $x_1^2+2x_2^2+2\sqrt{2}x_1x_3$.

(a) Find a matrix $\mathbf{A}$ such that the quadratic form $\mathbf{X}^T\mathbf{AX}$ with $\mathbf{X}=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$. (5 分)

(b) Use the principal axis theorem to find the standard form. (10 分)

5. Use the matrix exponential to solve the initial value problem $\mathbf{Y}'=\mathbf{AY}$, $\mathbf{Y}(0)=\mathbf{Y}_0$

where

$$\mathbf{A}=\begin{bmatrix}1&1&1\\1&0&1\\-1&-1&-1\end{bmatrix},\quad \mathbf{Y}_0=\begin{bmatrix}1\\1\\-1\end{bmatrix}.$$ (15 分)

6. Show that the given set $\mathbf{S}$ spans $\boldsymbol{R}^3$. If the set does not span $\boldsymbol{R}^3$, give a geometric description of the subspace that it does span.

(a) $\mathbf{S}=\{(1,\ 1,\ 1)^T,\ (1,\ 1,\ 0)^T,\ (1,\ 0,\ 0)^T\}$ (5 分)

(b) $\mathbf{S}=\{(1,\ 2,\ 4)^T,\ (2,\ 1,\ 3)^T,\ (4,\ -1,\ 1)^T\}$ (5 分)

(c) $\mathbf{S}=\{(1,\ 2,\ 3)^T,\ (0,\ 1,\ 2)^T,\ (-2,\ 0,\ 1)^T\}$ (5 分)

(d) $\mathbf{S}=\{(2,\ 1,\ -2)^T,\ (-2,\ -1,\ 2)^T,\ (4,\ 2,\ -4)^T\}$ (5 分)