國立臺北科技大學 111 學年度碩士班招生考試

系所組別：2401、2402、2403 光電工程系碩士班

第一節　工程數學　試題

注意事項：

1. 本試題共 7 題，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

(1) **(15%)** Let $\delta = \begin{bmatrix} 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a 3x3 matrix. Please compute $\exp(i\delta)$.

(2) **(15%)** Please use Laplace transform to solve the following initial value problem.

$y''-3y'+2y=4t-8, \quad y(0)=2, \quad y'(0)=7$

(3) **(15%)** Solve the following ordinary differential equation for $y(x)$.

$$\frac{d^2y}{dx^2}+4y=5xe^{-x}$$

(4) **(10%)** Solve the following ordinary differential equation for $y(x)$ by the method of separation of variables.

$$(x^3+17)\frac{dy}{dx}=x^2y$$

(5) **(15%)** Please find the Fourier transform of the Gaussian distribution

$$f(x)=exp\left(\frac{-x^2}{2a}\right).$$

(6) **(15%)** Please evaluate

$$\int_{-4}^{4}\int_{0}^{\sqrt{16-x^2}} exp\left(-(x^2+y^2)\right)dydx$$

(7) **(15%)** Let $\vec{F}=xy^2\hat{i}+yz^2\hat{j}+zx^2\hat{k}$ be a vector field. Compute the flux across the surface bounding the region defined by $2\le x^2+y^2+z^2\le 6$.