

## 國立臺北科技大學 111 學年度碩士班招生考試

系所組別：1120 機械工程系機電整合碩士班乙組

## 第一節 工程數學 試題

第 1 頁 共 1 頁

**注意事項：**

1. 本試題共 6 題，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Solve the following ordinary differential equation. (15%)

$$\left(1 + 2e^{\frac{x}{y}}\right) dx + 2e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$$

2. Show the following ODE is Non-exact. Using the Hint:  $\mu(x, y) = f(y)$  to solve the equation. (15%)

$$4ydx + (x + 12xy)dy = 0$$

3. Solve for the following differential equation. (15%)

$$y'' + y = x \cdot \sin x$$

4. Use the Laplace transform to solve the following equation. (15%)

$$y'' + 4y = f(t), \quad y(0) = y'(0) = 0,$$

$$f(t) = \begin{cases} 0, & t < 6 \\ t, & t \geq 6 \end{cases}$$

5. Find eigenvalues and eigenvectors of matrix. (15%)

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

6. Consider the temperature distribution  $u(x, t)$  along a thin, homogeneous bar of length  $L$ . The initial temperature function  $f(x)$  is a constant  $A$ . The both ends of the bar are kept at temperature zero. The boundary value problem modeling this

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} \quad \text{for } 0 < x < L, t > 0$$

$$u(0, t) = u(L, t) = 0, t \geq 0$$

$$u(x, 0) = f(x) = A$$

Please step by step solve for the function of the temperature distribution  $u(x, t)$  (25%)