111EE07

# 國立臺北科技大學 111 學年度碩士班招生考試

## 系所組別：2151 電機工程系碩士班戊組

## 第一節　線性代數　試題（選考）



**注意事項：**

1. 本試題共 8 題，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。
4. 各題均須完整推導，否則將酌予扣分。

1. (15%) Show that if two squared matrices **A** and **B** are similar, then they have the same eigenvalues.

2. (10%) Find a least-squares solution of **Ax** = **b** for

$$\mathbf{A}=\begin{bmatrix}1 & -6\\ 1 & -2\\ 1 & 1\\ 1 & 7\end{bmatrix} \text{ and } \mathbf{b}=\begin{bmatrix}-1\\ 2\\ 1\\ 6\end{bmatrix}$$

3. (10%) Consider an $M$x$N$ column-wise orthonormal matrix **A.** Prove or disprove the following statements.

   (1) (5%) $\mathbf{A}^H\mathbf{A}$ is an identity matrix.

   (2) (5%) $\mathbf{A}\mathbf{A}^H$ is an identity matrix.

4. (25%) Consider a squared matrix **H** with the singular value decomposition **H** = **ABC**, where **A** and **C** are unitary and **B** is diagonal.

   (1) (5%) Show that **H** is not invertible if one diagonal element of **B** is equal to zero.

   (2) (10%) Show that such decomposition is not unique if the diagonal elements of **B** are not necessary with decreasing order.

   (3) (5%) Show that $\|\mathbf{ACx}\|=\|\mathbf{x}\|$ for vector **x**.

   (4) (5%) Show that $\mathbf{A}^{-1}=\mathbf{A}^H$.

5. (10%) Show that if a vector set contains a zero vector, then the set is a linearly dependent set.

6. (10%) Explain the column space and null space of a matrix.

7. (10%) Assume the mapping defined by

$$T(a_0+a_1t+a_2t^2)=4a_0+(5a_0-2a_1)t+(4a_1+a_2)t^2$$

   is linear. Find the matrix representation of $T$ relative to the bases $B=\{1,t,t^2\}$.

8. (10%) For a linear system **Ax**=**b**, where **A** is an $N$x$N$ matrix and **b** is an $N$x1 vector. Assume that **A** and **b** are given. Explain in detail how to apply the QR decomposition of **A** so that **x** can be solved with back substitution.