

國立臺北科技大學 111 學年度碩士班招生考試

系所組別：2142 電機工程系碩士班丁組

第一節 線性代數 試題 (選考)

第 1 頁 共 1 頁

注意事項：

1. 本試題共 6 題，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。
4. 各題均須完整推導，否則將酌予扣分。

1. (20%) Consider a squared matrix \mathbf{A} as:

$$\mathbf{A} = \begin{bmatrix} 1 & -3 & 2 & -4 \\ 0 & 0 & 5 & -7 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- (1) (5%) Find a basis of the null space of \mathbf{A} .
- (2) (5%) Determine the rank of \mathbf{A} .
- (3) (5%) Find the determinant of \mathbf{A} .
- (4) (5%) Prove or disprove \mathbf{A} is invertible.

2. (25%) Consider a complex 4×4 full-rank matrix \mathbf{H} . The QR decomposition and singular value decomposition of \mathbf{H} are given by $\mathbf{H}=\mathbf{QR}$ and $\mathbf{H}=\mathbf{ABC}$, respectively. The diagonal elements of \mathbf{B} are $\{4,3,2,1\}$.

- (1) (10%) Show that $\|\mathbf{ACQx}\| = \|\mathbf{x}\|$ for vector \mathbf{x} .
- (2) (10%) Find the diagonal elements of \mathbf{R} if they are all non-negative and equal.
- (3) (5%) Show that $\mathbf{A}^{-1} = \mathbf{A}^H$

3. (15%)

- (1) (5%) Find a least-squares solution of $\mathbf{Ax} = \mathbf{b}$ for

$$\mathbf{A} = \begin{bmatrix} 1 & -2 \\ -1 & 2 \\ 0 & 3 \\ 2 & 5 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 3 \\ 1 \\ -4 \\ 2 \end{bmatrix}$$

- (2) (10%) Explain why we need the least squares solution.

4. (10%) Let \mathbf{A} be a complex squared matrix. Show that $\mathbf{Ax}=\mathbf{0}$ if and only if $\mathbf{A}^H \mathbf{Ax}=\mathbf{0}$.

5. (20%) Consider an $N \times N$ matrix \mathbf{A} with N independent columns.

- (1) (5%) Use the definition of linear independence to show that $\mathbf{Ax}=\mathbf{0}$ has only trivial solution.
- (2) (10%) Show that \mathbf{A} does not contain any zero vector.
- (3) (5%) Determine the dimension of the null space of \mathbf{A} .

6. (10%) Assume the mapping defined by

$$T(a_0 + a_1 t + a_2 t^2) = 2a_0 + (5a_0 - 2a_1)t + (4a_1 + a_2)t^2$$

is linear. Find the matrix representation of T relative to the bases $B = \{1, t, t^2\}$.