國立臺北科技大學 111 學年度碩士班招生考試

系所組別：2141 電機工程系碩士班丁組

第一節　機率　試題（選考）

注意事項：

1. 本試題共五題，每題 20 分，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、Assume X has PMF

$$P_X(x) = \binom{4}{x}(1/2)^4$$

Let $B = \{X \neq 0\}$

(一) Find $P(B)$. (10 分)

(二) Find $P_{X|B}(x)$. (10 分)

二、Let $X_1, X_2, X_3, \ldots$ denote a sequence of independent samples of a random variable X with variance $Var[X]$. Define a new random sequence $Y_1, Y_2, Y_3, \ldots$ as $Y_1 = X_1 - X_2$ and $Y_n = X_{2n-1} - X_{2n}$.

(一) Find $E[Y_n]$ and $Var[Y_n]$. (10 分)

(二) Find the expected value and variance of $M_n(Y) = \frac{Y_1+Y_2+\cdots+Y_n}{n}$. (10 分)

三、Integral circuits from a certain factory pass a certain quality test with probability 0.8. The outcomes of all tests are mutually independent.

(一) Find the expected number of tests necessary to find 500 acceptable circuits. (10 分)

(二) Find the probability of finding 500 acceptable circuits in a bath of 600 circuits. (10 分)

四、Let X be a Gaussian $(0, \sigma)$ random variable with the moment generating function $\phi_X(s) = e^{\sigma^2 s^2/2}$. Let Y be a Gaussian (μ, σ) random variable.

(一)Use the moment generating function to show that $E[X^2] = \sigma^2$. (10 分)

(二)Define $Y = X + \mu$, show that $E[Y^2] = \sigma^2 + \mu^2$. (10 分)

五、For random variable X and Y, we wish to use Y to estimate X using the form $\hat{X} = aY$.

(一)Find a^*, the value of a that minimizes the mean square error $e = E[(X - aY)^2]$. (10 分)

(二)For $a = a^*$, find the minimum mean square error e^*. (10 分)