

國立臺北科技大學 111 學年度碩士班招生考試

系所組別：2131 電機工程系碩士班丙組

第一節 工程數學 試題 (選考)

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注意事項：

1. 本試題共六題，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Consider the differential equation

$$(\cos(x+y) - \sin(x+y))dx - \sin(x+y)dy = 0.$$

(a) (5%) Try the integrating factor $\mu(x) = e^{ax}$ and find a .

(b) (10%) Find the general solution.

2. (20%) Solve the differential equation $(x+5)^2 y'' - 3(x+5)y' + 3y = x$, $x > 0$.

3. (15%) Let the Laplace transform of $f(t)$ be $F(s)$ which satisfies the

$$\text{differential equation } F''(s) + 5F'(s) + 6F(s) = 0; \quad F(0) = 1, F'(0) = 0.$$

Compute $f(t)$.

4. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 5 & 1 \\ 3 & 5 & 7 \end{bmatrix}$ with $UAV = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 6 \end{bmatrix}$, where U

and V are products of elementary matrices representing elementary row and column operations respectively.

(a) (10%) Find U and V , provided $\det(U) = 1$ and $\det(V) = 1$.

(b) (10%) Compute U^{-1} and V^{-1} .

5. (15%) Consider the 3×3 real matrix A with a characteristic equation $\lambda^3 - 4\lambda^2 + 5\lambda - 2 = 0$. Compute the matrix function e^A in the form of second-degree polynomial of A .

6. Consider the matrix $A = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$.

(a) (5%) Find the null space of A , $N(A)$.

(b) (5%) Find the orthogonal complement of $N(A)$.

(c) (5%) Show that the orthogonal complement obtained in (b) is a subspace of R^4 .