國立臺北科技大學 111 學年度碩士班招生考試

系所組別：1501、1502 自動化科技研究所

第一節　工程數學　試題

注意事項：

1. 本試題共 5 題，每題 20 分，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (20%) Prove or disprove the following statements in detail (do not only answer True or False, a proof or counter example is needed):

   (1) (5%) If A and B are two R^2 matrices. $(A+B)^{-1} = A^{-1} + B^{-1}$.

   (2) (5%) If A is an $n \times n$ orthogonal matrix, then rank $A = n$.

   (3) (5%) Consider $Ax = b$ where A is $m \times n$. If the rank of matrix A is n, then there is a solution.

   (4) (5%) $(4, 2, 3)^T$, $(2, 3, 1)^T$, and $(2, -5, 3)^T$ are linearly dependent.

2. (20%) In a certain town 30 percent of the married women get divorced each year and 20 percent of the single women get married each year. There are 8000 married women and 2000 single women and the total population remains constant. Find the number of married women and single women after 5 years. What will be the long-range prospects if these percentages of marriages and divorces continue indefinitely into the future?

3. (20%) For each matrix, find the characteristic equation, and the eigenvalues and associated eigenvectors

   (1) (10%)

   $$\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$

   (2) (10%)

   $$\begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$$

4. (20%) Find the solution of $y'' - 4y' + 3y = 4e^{3x}$, $y(0) = -1$, $y'(0) = 3$.

5. (20%) Solve the differential equation by Laplace transform.

   $y'' + y = \delta(x-1) \quad 0 < x < 2$

   The boundary conditions are $y(0)$=0 and $y(2)$=0.