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國立臺北科技大學 106 學年度碩士班招生考試

系所組別：2403 光電工程系碩士班

第二節　近代物理　試題　（選考）

注意事項：

1. 本試題共四題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

Planck constant is $h = 6.63 \times 10^{-34}$ *Js.*
Light speed $c = 3 \times 10^{8}$ *m/s*
$1\ eV = 1.6022 \times 10^{-19}\ J$
The condition for normalization $\int \Psi^* \Psi dx = 1$
$\sin^2 x = (1-\cos 2x)/2$
Electron mass $m_e = 9.11 \times 10^{-31}\ kg$

1. **(20 points)** For an infrared photon of wavelength $\lambda_0 = 1$ μm in free space:
(1) What is the frequency of the infrared photon? (5 points)
(2) What is the energy of the infrared photon? Please show your answer with the units of joule (J) (5 point) and electron volts (eV) (5 points).
(3) Please write down the conversion formula between wavelength (μm) and photon energy (eV). (5 points)
(4) As the photon energy of the infrared photon $\lambda_0 = 1$ μm contains the same amount of energy gained by the charge of a single electron moving across an electric potential, what is the electrical potential difference? (5 points)

2. **(10 points)** How many photons per second does a continuous wave He-Ne laser with the power of 1mW produce? The wavelength of He-Ne laser is 632.8 nm.

3. **(25 points)** For a plane-wave photon which has a known momentum with the fixed direction along z-axis and magnitude, the angular uncertainty σ_θ and the positional uncertainty σ_x are zero ($\sigma_\theta = 0$) and totally uncertain ($\sigma_x = \infty$), respectively. Therefore, it is equally likely to be detected anywhere in the *x-y* plane ($z = 0$).
(1) What is the uncertainty relation of the plane-wave propagating along z-axis? Note: the known position-momentum uncertainty relation is $\sigma_x \sigma_{kx} \geq 1/2$. Please calculate the uncertainty relation for the plane-wave photon in terms of the positional uncertainty σ_x and the angular uncertainty σ_θ about the z-axis (which is assumed <<1). (Hint: $\sigma_x \sigma_\theta \geq$?) (15 points)
(2) As the plane-wave photon passes through a rectangular aperture with the height and width D_x and D_y, the position of the photon becomes localized at the expense of a spread in the direction of its momentum. Can you prove that the angular divergence of the diffracted light given by $\theta_x = \lambda/D_x$ and $\theta_y = \lambda/D_y$ also follow the uncertainty relation? (10 points)

4. **(45 points)** Assume the potential U(x) in the time-independent Schrodinger equation to be zero inside a one-dimensional (1D) box of length L and infinite outside the box. For a particle with the mass m inside the box, a free particle wavefunction is appropriate. Since the probability of finding the particle outside the box is zero, the wavefunction must go to zero at the walls.
(1) Please write down the time-independent Schrodinger Equation? (10 points) Please clarify every variable that you use.
(2) The form of the wavefunction for a particle inside the 1D box of length L is $\psi(x) = A\sin(kx)$. What is the form of k (10 points) and A (10 points) in terms of the length L of the 1D box?
(3) For an electron in carbon atom with a diameter 0.182 nm, what is its first ionization energy (eV)? Hint: Consider as a 1D box with the length of 0.182 nm and the first ionization energy, the same with the ground state energy. (15 points)