106 EE 07

國立臺北科技大學 106 學年度碩士班招生考試

系所組別：2143 電機工程系碩士班丁組

第一節　機率　試題　（選考）

第一頁　共一頁

注意事項：

1. 本試題共五題，每題 20 分，共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一. Consider a transmission system that in each transmission a bit is sent. The transmitted bit is decoded as 0 or 1 at the receiver, with error probability p. Assume the error probability is independent for each transmission.

1. If the transmission continues until the receiver makes its first error, what is the PMF of X, the number of bits transmitted? (10%)
2. If the transmission system totally transmits 100 bits, what is the PMF of Y, the number of error bits in the 100 transmitted bits? (10%)

二. Consider the following conditional PMF

$$P_{X|H}(x)=\begin{cases}0.1(0.9)^{x-1}, & x=1,2,3,\ldots\\ 0, & \text{otherwise}\end{cases}$$

$$P_{X|N}(x)=\begin{cases}0.05(0.95)^{x-1}, & x=1,2,3,\ldots\\ 0, & \text{otherwise}\end{cases}$$

1. Assume P[H]=0.4 and P[N]=0.6. Find $P_X(x)$. (10%)
2. Find E[X]. (10%)

三. Consider a continuous random variable X with PDF $f_X(x)$ and CDF

$$F_X(x)=\int_{-\infty}^{x} f_X(u)du$$

1. Show that $f_X(x)\geq 0$ for all x. (10%)
2. Show that P[X=a] = 0, for any constant a. (10%)

四. Assume X is an exponential (λ) random variable. Let $K=\lceil X\rceil$ where $\lceil . \rceil$ is an ceiling operator (for example, $\lceil 3.2\rceil=4$).

1. Find PMF $P_K(k)$ in terms of CDF $F_X(x)$. (10%)
2. Show that K is a geometric (p) random variable with $p=1-e^{-\lambda}$ (10%)

五. Assume the joint CDF of X and Y is

$$F_{X,Y}(x,y)=\begin{cases}0, & x<5\\ 0, & y<6\\ (x-5)(y-6), & 5\leq x<6,\ 6\leq y<7\\ y-6, & x\geq 6,\ 6\leq y<7\\ x-5, & 5\leq x<6,\ y\geq 7\\ 1, & \text{otherwise}\end{cases}$$

1. Find $F_X(x)$. (10%)
2. Show that $P[x_1\leq X<x_2, y_1\leq Y<y_2]=F_{X,Y}(x_2,y_2)-F_{X,Y}(x_2,y_1)-F_{X,Y}(x_1,y_2)+F_{X,Y}(x_1,y_1)$. (10%)