107EE07

國立臺北科技大學 107 學年度碩士班招生考試

系所組別：2143 電機工程系碩士班丁組

第一節　機率　試題　(選考)

第一頁　共一頁

注意事項：
1. 本試題共五題，每題 20 分，共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一. Consider the length of a video X in minutes distributed from a website has the following probability mass function (PMF):

$$P_X(x) = \begin{cases} 0.15 & x = 1,2,3,4, \\ 0.1 & x = 5,6,7,8, \\ 0 & otherwise \end{cases}$$

Given the event that $x \in L = \{5,6,7,8\}$:

1. (5%) Find the conditional PMF $P_{X|L}(x)$.
2. (5%) Find the conditional expected value $\mathrm{E}[X \mid L]$.
3. (5%) Find the conditional variance $\mathrm{Var}[X \mid L]$.
4. (5%) Find the conditional standard deviation $\sigma_{X|L}$.

二. Let σ_X^2 and σ_Y^2 denote the variances of random variable X and Y. Let $W = X - aY$, where a is a constant.

1. (10%) Find $\mathrm{Var}[W]$, and show that $\mathrm{Cov}[X,Y] \le \dfrac{1}{2a}\mathrm{Var}[X] + \dfrac{a}{2}\mathrm{Var}[Y]$.
2. (10%) Show that the correlation coefficient of the two random variable X and Y, $\rho_{X,Y} = \dfrac{\mathrm{Cov}[X,Y]}{\sigma_X \sigma_Y}$, has the property that $-1 \le \rho_{X,Y} \le 1$.

三. Consider random variable X and Y have the joint probability density function (PDF)

$$f_{X,Y}(x,y) = \begin{cases} \lambda\mu e^{-(\lambda x + \mu y)}, & x \ge 0,\ y \ge 0 \\ 0 \quad , & \text{otherwise} \end{cases}$$

where λ and μ are constants.

1. (10%) Let $W = Y / X$. Find the PDF of $f_W(w)$.
2. (10%) Let $Z = aX$ where a is a constant. Find the PDF of $f_X(x)$ and $f_Z(z)$.

四. For a random variable X, the moment generating function (MGF) of X is defined as

$$\phi_X(s) = \mathrm{E}[e^{sX}].$$

1. (10%) Show that the n-th moment of the random variable X, $\mathrm{E}[X^n] = \left.\dfrac{d^n \phi_X(s)}{ds^n}\right|_{s=0}$.
2. (10%) Suppose $\phi_X(s) = 10/(10-s)$. Find $\mathrm{E}[X]$.

五. Given $X_1, X_2, \ldots\ldots X_n$ a sequence of iid random variables with expected value μ_X and variance σ_X^2. Let $Z_n = \dfrac{\sum_{i=1}^{n}(X_i - n\mu_X)}{\sqrt{n\sigma_X^2}}$.

1. (10%) Find $\mathrm{E}[Z_n]$ and $\mathrm{Var}[Z_n]$.
2. (10%) Show that when n is approaching to ∞, the cumulative distribution function (CDF) of random variable Z, $F_Z(z)$, is approaching to the standard normal CDF, $\phi(z)$. In other words, $\lim_{n\to\infty} F_Z(z) = \phi(z)$.