國立臺北科技大學 108 學年度碩士班招生考試

系所組別：1201 製造科技研究所

第一節　微分方程　試題（選考）

注意事項：

1. 本試題共 6 題，每題 15-20 分，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

---

1. Use both (a) separable variable method (10pts) and (b)integrating factor method (10pts) to solve

$$y' = xy$$

(20pts)

2. Solve the following differential equation:

$$y' + \frac{1}{3}y = \frac{1}{3}(1-2x)y^4$$

(15pts)

3. Solve the following second order non-homogeneous equation:

$$y'' - 4y' - 12y = te^{4t}$$

(15pts)

4. Solve the following differential equation

$$y'' + 4y = f(t)$$

$$y(0) = y'(0) = 0 \qquad f(t) = \begin{cases} 0 \ , \ t < 3 \\ t \ , \ t \geq 3 \end{cases}$$

(15pts)

5. Solve the following differential equation

$$x^2 y'' + x\left(\frac{1}{2} + 2x\right)y' + \left(x - \frac{1}{2}\right)y = 0$$

(15pts)

6. Consider an elastic string of length L, fastened at its ends on the x axis at $x=0$ and $x=L$. The string is displaced, then released from rest to vibrate in the x, y plane. We want to find the displacement function $y(x, t)$, whose graph is a curve in the x, y plane showing the shape of the string at time t. If we took a snapshot of the string at time t, we would see this curve.

The boundary value problem for this displacement function is

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \text{ for } 0 < x < L, t > 0$$

$$y(0,t) = y(L,t) = 0 \text{ for } t \geq 0,$$
$$y(x,0) = f(x) = 0 \text{ for } 0 \leq x \leq L,$$
$$\frac{\partial y}{\partial t}(x,0) = 0, \text{for } 0 \leq x \leq L,$$

Find the Fourier solutions of this wave equation.

(20pts)