EE08

國立臺北科技大學108學年度碩士班招生考試

系所組別：2150 電機工程系碩士班戊組

第一節　線性代數　試題

注意事項：

1. 本試題共10題，每題10分，共100分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，**解題過程要詳細列出來**，否則不予計分。

1. Solve (if possible) the following system of equations using ***Gauss-Jordan*** elimination.

$$\begin{cases} x_1 + 3x_2 + 10x_3 = 18 \\ -2x_1 + 7x_2 + 32x_3 = 29 \\ -x_1 + 3x_2 + 14x_3 = 12 \\ x_1 + x_2 + 2x_3 = 8 \end{cases}$$ (10%)

2. Consider a linear system whose augmented matrix is of the form

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & a & b \end{array}\right]$$

(a) For what values of a and b will the system have infinitely many solution?(5%)
(b) For what values of a and b will the system be inconsistent? (5%)

3. Let $E = [v_1, v_2, v_3] = [(1, 1, 1)^T, (2, 3, 2)^T, (1, 5, 4)^T]$
$F = [u_1, u_2, u_3] = [(1, 1, 0)^T, (1, 2, 0)^T, (1, 2, 1)^T]$
Find the transition matrix from E to F.
If $x = 3v_1 + 2v_2 - v_3$, find the coordinate of x with respect to the ordered basis F. (10%)

4. The Fibonacci sequence is defined by the difference equation $a_n = a_{n-1} + a_{n-2}$, with initial conditions $a_1 = 1$, $a_2 = 1$. Solve this difference equation. Show that $\frac{a_n}{a_{n-1}}$ approaches the number $\frac{1+\sqrt{5}}{2}$ as n increases. (10%)

5. Find a ***QR*** factorization of the matrix $\begin{bmatrix} 1 & 2 & 5 \\ -1 & 1 & -4 \\ -1 & 4 & -3 \\ 1 & -4 & 7 \\ 1 & 2 & 1 \end{bmatrix}$. (10%)

6. Consider the vector space C **[-1, 1]** with inner product defined by $< f, g > = \int_{-1}^{1} f(x)g(x)\,dx$, find an orthonormal basis for the subspace spanned by $\mathbf{1}, x, x^2$, and x^3. (10%)

7. Let $T : R^5 \rightarrow R^4$ be the linear transformation given by $T(x) = Ax$, where x is in R^5 and $A = \begin{bmatrix} 1 & 2 & 0 & 1 & -1 \\ 2 & 1 & 3 & 1 & 0 \\ -1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 2 & 8 \end{bmatrix}$. Find a basis for the kernel of T as a subspace of R^5. (10%)

8. Find an ***LU*** factorization of $B = \begin{bmatrix} 2 & -4 & -2 & 3 \\ 6 & -9 & -5 & 8 \\ 2 & -7 & -3 & 9 \\ 4 & -2 & -2 & -1 \\ -6 & 3 & 3 & 4 \end{bmatrix}$. (10%)

9. Find a singular value decomposition (***SVD***) of $C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ (10%)

10. Analyze the following equation. Sketch its graph. (10%)

$$x^2 - 6xy + y^2 - 8 = 0$$