EE07

國立臺北科技大學 108 學年度碩士班招生考試

系所組別：2143 電機工程系碩士班丁組

第一節　機率　試題（選考）

注意事項：

1. 本試題共五題，每題 20 分，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一. A continuous random variable X has the probability density function

$$f(x)=\begin{cases}0, & x<0\\ 1/5, & 0\le x<k\\ 4/5, & k\le x<2\\ 0, & x>2\end{cases}$$

1. (10%) Find the value of k which make $f(x)$ a valid probability density function.
2. (10%) Find the cumulative distribution function $F(x)$.

二. Let X and Y are continuous random variables with the joint probability density function

$$f(x,y)=\begin{cases}\frac{x+y+1}{5}, -1\le x<1 \ \ and \ \ 1\le y<2\\ 0, \qquad\qquad otherwise\end{cases}$$

1. (10%) Find the probability $P(Y<1.5 \mid X=0)$.
2. (10%) Are the random variables X, Y independent ? Justify your answer.

三. Write 'True' or 'False' and justify your answer:

1. (5%) Let A and B be events. $P(B|A)=P(A|B)\frac{1-P(B^C)}{1-P(A^C)}$.
2. (5%) Support A and B are disjoint events. If A and B are independent, then either $P(A)=0$ or $P(B)=0$.
3. (5%) Let X and Y are random variables. $E[(X+Y)^2]=E[X^2]+2E[X]E[Y]+E[Y^2]$.
4. (5%) $P(X^2\ge 1)\le E[X^2]$.

四. Gaussian random variables and central limit theorem:

1. (5%) Let X and Y be independent Gaussian random variables with zero mean and unit variance. Find $E[X^8Y^{12}]$.
2. (5%) If X and Y are jointly Gaussian random variables with $E[X]=0, E[Y]=-1$, $Var[X]=1$, $Var[Y]=2$ and $Cov(X,Y)=1$. Let $W=X-Y$. Find $Var[W]$.
3. (5%) Let X be Gaussian random variable with $E[X]=2$ and $E[X^2]=9$. Find $P(2\le X<9)$. Write your answer only in terms of the standard normal CDF $\Phi(z)$.
4. (5%) Suppose you toss a fair coin one million times. Using the central limit theorem to calculate the probability that it falls on heads more than 600,000 times.

五. Let X be the sent bit and Y the received bit in a binary communication channel. The joint probability function $P(x,y)$ is given as

$P(x,y)$	$x=0$	$x=1$
$y=0$	0.4	0.1
$y=1$	0.1	0.4

1. (10%) The bit transmission is said to have an error if the sent bit and the revived bit are different. In other words, $P(error)=P(Y\neq X)$. Find $P(error)$.
2. (10%) If an 8-bit word is sent over this binary communication channel, what is the probability that 1-bit or less errors occurs?