EE06

國立臺北科技大學 108 學年度碩士班招生考試

系所組別：2142 電機工程系碩士班丁組

第一節　訊號與系統　試題（選考）

注意事項：

1. 本試題共 9 題，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。
4. 答案需合理之數學推導，否則酌予扣分。

1. (5%) Determine the convolution $y(t) = x(t) * h(t)$ of the following signal pair:

$$x(t) = e^{-2t}u(t) \text{ and } h(t) = u(t)$$

where $u(t)$ is the unit step function.

2. (10%) Show that $x(t) = e^{j\omega_0 t}$ is periodic with period T.

3. (15%) Let $X(e^{j\omega})$ denote the Fourier transform of discrete-time signal $x[n]$.
   (a) (8%) Show that $X(e^{j\omega})$ is always periodic in ω with period 2π.
   (b) (7%) Determine $X(e^{j\omega})$ if $x[n] = e^{j2n}$.

4. (10%) Find the Fourier transform representations of the following signals:
   (a) (5%) $x(t) = \sum_{k=-\infty}^{\infty} \delta(t-9k)$.
   (b) (5%) $x(t) = e^{-7t}u(t)$.

5. (5%) Prove that if $x(t)$ is real, then its spectrum $X(j\omega)$ will be conjugate symmetric.

6. (10%) Let $X(s = \sigma + j\omega)$ be the Laplace transform of $x(t)$.
   (a) (5%) Show that $X(s)$ exactly equals the Fourier transform of $x(t)e^{-\sigma t}$.
   (b) (5%) Determine the Laplace transform of $x(t) = e^{-2t}u(t)$.

7. (10%) Show that $x[n] = a^n u[n]$ and $y[n] = -a^n u[-n-1]$ have the same z-transform expression but different regions of convergence.

8. (15%) Let $\cos(\omega_c t)$ be a carrier signal and consider a baseband signal $x(t)$ with spectrum

$$X(j\omega) = \begin{cases} 2A & |\omega| < \omega_B \ll \omega_c \\ 0 & |\omega| > \omega_B \end{cases}.$$

   (a) (5%) Derive the Fourier transform of $\cos(\omega_c t)$.
   (b) (5%) Consider amplitude modulation (AM) $r(t) = x(t)\cos(\omega_c t)$. Sketch the spectrum of $r(t)$.
   (c) (5%) Detail the synchronous demodulation for the AM.

9. (20%) Consider a band-limited signal $x(t)$ with spectrum

$$X(j\omega) = \begin{cases} 1 - \dfrac{\omega}{\omega_M} & 0 \le \omega \le \omega_M \\ 1 + \dfrac{\omega}{\omega_M} & -\omega_M \le \omega < 0 \\ 0 & otherwise \end{cases}$$

Let $p(t)$ be a sampling function expressed as

$$p(t) = T\sum_{k=-\infty}^{\infty} \delta(t-kT).$$

   (a) (5%) Sketch $X(j\omega)$.
   (b) (5%) Sketch the spectrum of $y(t) = x(t)p(t)$.
   (c) (5%) Determine the maximum T given that $x(t)$ is recoverable (no aliasing effect).
   (d) (5%) Detail how to perfectly reconstruct $x(t)$ by using $y(t)$.