國立臺北科技大學 108 學年度碩士班招生考試

系所組別：1501、1502、1503、1504

自動化科技研究所

第一節　工程數學　試題

注意事項：

1. 本試題共 6 題，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

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1. Please solve the following initial value problems.

    (a) $\dot{y} = \dfrac{2y}{x} + x^2 e^x$, $y(2) = 0$ (10%)

    (b) $\dot{y} = -3x^2 y + x e^{-x^3}$, $y(0) = -1$ (10%)

2. Please find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$. (12%)

3. Compute A^{50}, where $A = \begin{bmatrix} -3 & 2 \\ 1 & -4 \end{bmatrix}$. (15%)

4. (a) Find the Fourier series of $f(t) = t^2$, $-1 \le t \le 1$ with period $p = 2L = 2$. (15%)

    (b) Based on the results in (a), please show that

    $$1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} \cdots = \frac{\pi^2}{12}.$$ (5%)

5. Solve the following differential equations by Laplace transform

    $\ddot{y} + 9y = 1 + t$, $y(0) = -1$, $\dot{y}(0) = 1$. (15%)

6. Please solve the following differential equation $\ddot{y} - 3y' + 2y = \sin(e^{-x})$. (18%)