109RA06

國立臺北科技大學 109 學年度碩士班招生考試

系所組別：1422 能源與冷凍空調工程系碩士班乙組

第二節　流體力學　試題（選考）

注意事項：

1. 本試題共 5 題，每題 20 分，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. A laminar flow in the entrance to a pipe with radius r_o is shown in Fig. 1. The entrance velocity distribution is uniform, $u=U_o$, and the velocity distribution downstream is parabolic in profile, $u(r)=C(r_o^2-r^2)$. Assume that the pressure distribution at location o is P_o and at location x is P_x. P_o and P_x are constant. Please determine
   (a) the value of C. (5%)
   (b) the viscous drag force exerted on the pipe walls between o and x. (15%)

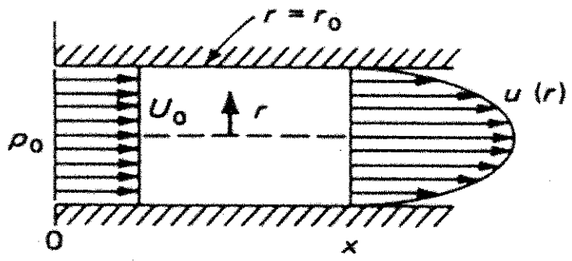

Fig. 1

2. As shown in Fig. 2, a tornado may be simulated as a Rankine vortex which has a two-port circulating flow in plane polar coordinates, with $u_\theta = U$ at $r = R$ and $u_r = 0$. For a Rankine vortex, the velocity distribution is as follows:

$u_\theta = \frac{U}{R} r$, $u_r=0$ if $r \le R$

$u_\theta = \frac{C}{r}$, $u_r=0$ if $r > R$

where C is a constant. Please determine
(a) constant C. (2%)
(b) an expression of stream function φ for a Rankine vortex. (12%)
(c) the vorticity in each part of the Rankine vortex. (6%)

The relation between velocity components and stream function is expressed by $u_\theta = -\frac{\partial \varphi}{\partial r}$, $u_r = \frac{1}{r}\frac{\partial \varphi}{\partial \theta}$. The vorticity component about the z-axis is $\omega_z = \frac{1}{r}\frac{\partial}{\partial r}(ru_\theta) - \frac{1}{r}\frac{\partial u_r}{\partial u_\theta}$

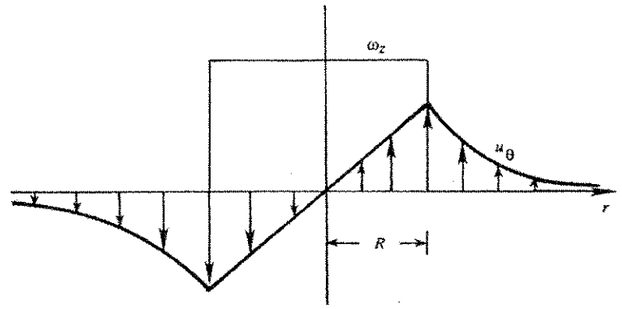

Fig. 2

3. Consider the laminar flow of a fluid layer falling down a plane inclined at an angle α with the horizontal, as depicted in Fig. 3. Assume that air resistance is negligible at the free surface. If h is the thickness of the layer in the fully developed stage, please
   (a) determine the velocity distribution. (10%)
   (b) find the volume flow rate per unit width. (4%)
   (c) find the frictional stress on the wall. (4%)
   (d) plot the distribution of shear stress between the bottom wall and free surface. (2%)

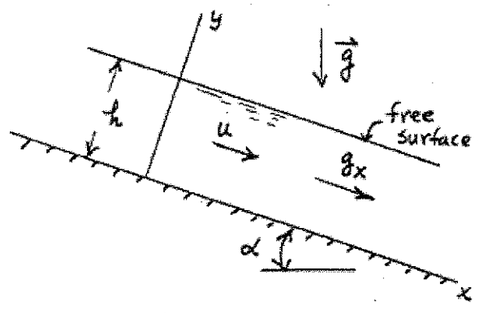

Fig. 3

4. Consider a two-dimensional laminar boundary-layer flow over a flat plate. Assume that the velocity profile is given by

$\frac{u}{U} = a + by + cy^2$

where U is the free stream velocity.
(a) Determine the constants a, b and c. (8%)
(b) Show that $\frac{\delta}{x} = \sqrt{\frac{30}{\mathrm{Re}_x}}$, where δ is the boundary-layer thickness and $\mathrm{Re}_x = \frac{Ux}{\nu}$. (12%)

注意：背面尚有試題

5. As shown in Fig. 4, a tank with fixed volume, ∀, contains brine with initial density, ρ_i, greater than water density, ρ_{H_2O}. Pure water enters the tank steadily and mixes thoroughly with the brine in the tank. The liquid level in the tank remains constant. Derive expressions for (a) the rate of change of density for the liquid mixture in the tank, and (b) the time required for the density to reach the value ρ_f, where $\rho_i > \rho_f > \rho_{H_2O}$. (20%)

Note: A first-order linear differential equation is given by $y'(x) + p(x)y = q(x)$. Its solution is

$$y(x) = e^{-\int p(x)dx}\int\left(q(x)e^{\int p(x)dx}\right)dx + Ce^{-\int p(x)dx}$$

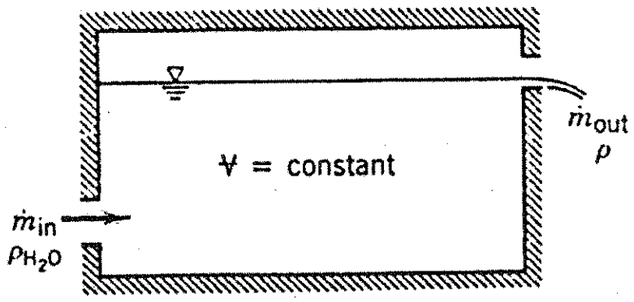

Fig. 4