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國立臺北科技大學 109 學年度碩士班招生考試

系所組別：4100 工業工程與管理系碩士班

第一節 統計學 試題

第 1 頁 共 3 頁

注意事項：

1. 本試題選擇題共 20 題，每題 3 分，計算題共 4 題，每題 10 分，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、單選題（共 60 分，每題 3 分）

1. If X and Y are independent random variables with variances $\sigma_X^2 = 5$ and $\sigma_Y^2 = 3$, the variance of the random variable $Z = -2X + 4Y - 3$ is (A) 28 (B) 68 (C) 77 (D) 19.
2. If X_1, X_2, \dots, X_n are independent random variables having identical normal distributions with mean μ and variance σ^2 , then the random variable $Y = \sum_{i=1}^n \left(\frac{x_i - \mu}{\sigma}\right)^2$ (A) has a chi-squared distribution with n degrees of freedom. (B) has a t-distribution with $n-1$ degrees of freedom. (C) has a standard normal distribution. (D) has an F-distribution with $n-1$ degrees of freedom.
3. As the degree of freedom increase, a t-distribution approximates (A) a normal distribution. (B) a chi-square distribution. (C) a F distribution of variance. (D) a standard normal distribution.
4. A nonparametric test for the equivalence of two populations would be used instead of a parametric test for the equivalence of the population parameters if (A) no information about the populations is available. (B) the samples are not independent. (C) the samples are very large. (D) the parametric test fails to reject the null hypothesis.
5. If $P(A \cup B) = 2/3$ and $P(A^c | B^c) = 1/2$, then $P(B) =$ (A) 1/2 (B) 1/3 (C) 1/5 (D) 1/6.
6. If $X_1 \sim N(\mu_1, \sigma_1)$ and $X_2 \sim N(\mu_2, \sigma_2)$ are independent random variables, then which of the following is not true? (A) $X_1 - X_2$ has a normal distribution (B) $2X_1 - X_2$ has a normal distribution (C) $X_1^2 + X_2^2$ has a $\chi^2_{v=2}$ distribution (D) $\left(\frac{x_1 - \mu_1}{\sigma_1}\right)^2 + \left(\frac{x_2 - \mu_2}{\sigma_2}\right)^2$ has a Gamma($\alpha=1, \beta=2$) distribution.

7. To test $H_0: \mu = 5.5$ against $H_a: \mu < 5.5$ at the 0.05 level of significance, how large a sample (n) is required if the power $(1-\beta)$ of the test is to be 0.95 when $|\delta|/\sigma < 0.6$? (A) $n < 10$ (B) $10 \leq n < 20$ (C) $20 \leq n < 30$ (D) $n \geq 30$
8. General Hospital has noted that they admit an average of 7 patients per hour. What is the probability that during the next two hours exactly 7 patients will be admitted? (A) 0.0174 (B) 0.0222 (C) 0.1126 (D) 0.0009
9. X is a random variable that has an exponential distribution with parameter $\lambda = 1$. If $P(k < X < 4) = 0.117$ when the, find that $k =$ (A) 1.4 (B) 2.0 (C) 2.6 (D) 3.2.
10. The time it takes a worker on an assembly line to complete a task is exponentially distributed with a mean of 8 minutes. What is the probability that it will take a worker less than 4 minutes to complete the task? (A) 1.0000 (B) 0.3675 (C) 0.3935 (D) 0.7513
11. You have two lightbulbs for a particular lamp. Let X = the lifetime of the first bulb and Y = the lifetime of the second bulb (in unit of 1000 hours). Suppose that X and Y are independent and that each has an exponential distribution with parameter $\lambda = 0.5$. What is the probability that the total lifetime is between 1 and 2? (A) 0.4325 (B) 0.3387 (C) 0.2387 (D) 0.1740
12. Eleven percent of all students at a large university are absent on Mondays. If a random sample of 12 names is called on a Monday, what is the probability that four students are absent? (A) 0.0285 (B) 0.5623 (C) 0.0065 (D) 0.0114
13. If a random variable X has the moment generating function $M_X(t) = 1/(1-t^2)$, then the variance of X equals to (A) 0 (B) 1 (C) 2 (D) 4.
14. A random variable $X \sim \chi^2_{v=3}$, then a random variable $Y = 2X$ has an variance (σ^2) of (A) 4 (B) 6 (C) 12 (D) 24
15. A uniformly distributed probability density function ranging from 2 to 6 is (A) 0.25 (B) undefined (C) any value between 0 and 1 (D) 4
16. A population has a mean of 400 and a standard deviation of 25. A sample of 100 observations will be taken. The probability that the sample mean will be greater than 405 is (A) 0.9772 (B) 0.0228 (C) 0.4207 (D) 0.3805.
17. A fair die is thrown until the sum of the results of the throws exceeds 6. The random variable X is the number of throws needed for this. Let f be the distribution function of X , then $f(2) =$ (A) 0.333 (B) 0.417 (C) 0.583 (D) 0.722.

注意：背面尚有試題

18. The test for χ^2 goodness of fit (A) is always a lower tail test. (B) is always an upper tail test. (C) is always a two-tailed test. (D) can be a lower or an upper tail test.
19. To compute an interval estimate for the difference between the means of two populations, the t-distribution (A) is not restricted to small sample situations. (B) is restricted to small sample situations. (C) can be applied only when the populations have equal means. (D) can be applied only when the populations have equal standard deviations.
20. A criminologist conducted a survey to determine whether the incidence of certain types of crime varied from one part of a large city to another. The particular crimes of interest were assault, burglary, larceny, and homicide. The following table shows the numbers of crimes committed in four areas of the city during the past year. If we want to test if the occurrence of these types of crime is dependent on the city district, what statistical test should we use? (A) one-way ANOVA (B) two-way ANOVA (C) χ^2 independent test (D) two-way ANOVA without replicates

District	Type of Crime			
	Assault	Burglary	Larceny	Homicide
1	162	118	451	18
2	310	196	996	25
3	258	193	458	10
4	280	175	390	19

二、計算題（共 40 分，每題 10 分）

1. In order to estimate the difference between the average daily sales of two branches of a department store, the following data has been gathered.

Downtown Store North Mall Store

Sample size $n_1 = 23$ days $n_2 = 26$ days

Sample mean (in \$1000) $\bar{x}_1 = 37$ $\bar{x}_2 = 34$

Sample standard deviation (in \$1000) $s_1 = 4$ $s_2 = 5$

- (A) Determine the point estimate of the difference between the population means. (2 分)
- (B) Assume unequal variances, determine the degrees of freedom for an interval estimation. (3 分)
- (C) Compute the margin of error. (3 分)
- (D) Develop a 95% confidence interval for the difference between the two population means. (2 分)

2. Random variables Z_1, Z_2, \dots, Z_{10} are all independent and $\sim N(0,1)$, let $X = \sum_{i=1}^{10} Z_i^2$ and $Y = 3X$,
- (A) Find $P(3.9 < X < 6.7)$ (4 分)
- (B) Find $E(Y)$ (3 分)
- (C) Find $\text{Var}(Y)$ (3 分)

3. A Poisson model was hypothesized for the distribution of the number of genetic exchanges. Test the fit of a Poisson distribution to the data by estimating λ using $\alpha = 0.05$.

x = Number of Exchanges	0	1	2	3	4	5	6	7	8	9
Observed Counts	6	24	42	59	62	44	41	14	6	2

4. Suppose that X and Y are two random variables having the joint probability distribution $f(x,y)$. Find

- (A) $E(X)$ and $E(Y)$ (2 分)
- (B) $\text{Var}(X)$ and $\text{Var}(Y)$ (2 分)
- (C) $E(2X - 3Y)$ (2 分)
- (D) $E(XY^2)$ (2 分)
- (E) $\text{Cov}(X,Y)$ (2 分)

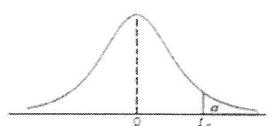
v	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.75	0.70	0.50	α
1	0.04303	0.03157	0.03628	0.03982	0.00393	0.0158	0.0642	0.102	0.148	0.455	
2	0.01000	0.0201	0.0404	0.0506	0.103	0.211	0.446	0.575	0.713	1.386	
3	0.0717	0.115	0.185	0.216	0.352	0.584	1.005	1.213	1.424	2.366	
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	1.923	2.195	3.357	
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	2.675	3.000	4.351	
6	0.676	0.872	1.131	1.237	1.635	2.204	3.070	3.455	3.828	5.348	
7	0.989	1.239	1.561	1.690	2.167	2.833	3.822	4.255	4.671	6.346	
8	1.344	1.617	2.002	2.180	2.733	3.490	4.594	5.071	5.527	7.344	
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.809	6.393	8.343	
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342	
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341	
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340	
13	3.565	4.107	4.765	5.009	5.892	7.041	8.634	9.290	9.926	12.340	
14	4.075	4.660	5.308	5.629	6.571	7.790	9.467	10.165	10.821	13.339	
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.037	11.721	14.339	
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338	
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338	
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338	
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338	
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337	
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.314	17.182	20.337	
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.210	18.101	21.337	
23	9.260	10.196	11.293	11.689	13.091	14.848	17.187	18.137	19.021	22.337	
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337	
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.867	24.337	
26	11.160	12.198	13.400	13.844	15.379	17.292	19.820	20.843	21.792	25.336	
27	11.808	12.878	14.125	14.573	16.151	18.114	20.703	21.749	22.719	26.336	
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.336	
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336	
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336	
31	22.164	23.838	24.433	26.509	29.051	32.345	33.06	34.872	39.335		
32	27.971	31.664	32.357	34.764	37.689	41.449	42.942	44.313	49.335		
33	35.534	37.485	39.699	40.482	43.188	46.459	50.641	52.294	53.809	59.335	

Table A.5 Critical Values of the Chi-Squared Distribution



Table A.3 (continued) Areas under the Normal Curve

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8613	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9993	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998

Table A.4 Critical Values of the *t*-Distribution

<i>v</i>	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.52					

