

109 EE 06

國立臺北科技大學 109 學年度碩士班招生考試

系所組別：2151 電機工程系碩士班戊組

第一節 線性代數 試題 (選考)

第 1 頁 共 1 頁

**注意事項：**

1. 本試題共 8 題，總計 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，解題過程要詳細列出來，否則不予計分。

1. Apply elementary row operations to transform the following matrix first into echelon form and then into reduced echelon form. (10 分)

$$\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

2. Determine all solutions of  $A\mathbf{x} = \mathbf{b}$ , where  $A = \begin{bmatrix} 3 & 5 & -4 \\ -3 & -2 & 4 \\ 6 & 1 & -8 \end{bmatrix}$  and  $\mathbf{b} = \begin{bmatrix} 7 \\ -1 \\ -4 \end{bmatrix}$ . (10 分)

3. Find the inverse of the matrix  $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$ , if it exists. (10 分)

4. Let  $B$  be the  $n \times n$  matrix, compute  $\det(B)$ . (10 分)

$$B = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 2 & \cdots & 2 \\ 1 & 2 & 3 & \cdots & 3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & 3 & \cdots & n \end{bmatrix}$$

5. Let  $A = \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{bmatrix}$ . Analyze the long-term behavior of the dynamical system defined by  $\mathbf{x}_{k+1} = A\mathbf{x}_k$  ( $k = 0, 1, 2, \dots$ ), with  $\mathbf{x}_0 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$ . (15 分)

6. Let  $\mathbf{b}_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$ ,  $\mathbf{b}_2 = \begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix}$ ,  $\mathbf{b}_3 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ , and  $\mathbf{x} = \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix}$ . Show that the set  $B = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$  is a basis of  $\mathbb{R}^3$ . Find  $[\mathbf{x}]_B$ , for the  $\mathbf{x}$  given above. (15 分)

7. Determine the kernel and the range of the transformation defined by the following matrix. (15 分)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 1 & 1 & 4 \end{bmatrix}$$

8. Find a set of orthonormal basis which is orthogonal to the vector  $(1, -2, -3)$ . (15 分)