

109 EE05

# 國立臺北科技大學 109 學年度碩士班招生考試

系所組別：2140 電機工程系碩士班丁組

## 第一節 機率 試題

第 1 頁 共 1 頁

**注意事項：**

1. 本試題共 5 題，每題 20 分，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一. Random variables  $X$  and  $Y$  have joint PDF

$$f_{X,Y}(x,y) = \begin{cases} k + 3x^2 & -0.5 < x < 0.5; -0.5 < y < 0.5 \\ 0 & \text{otherwise} \end{cases}$$

- (一) What is  $k$ ? (5%)
- (二) What is the marginal PDF of  $X$ ? (5%)
- (三) Are  $X$  and  $Y$  independent? (10%)

二. Random variables  $X$  and  $Y$  have joint PMF

$P_{X,Y}(x,y)$	$y = -3$	$y = -1$	$y = 1$	$y = 3$
$x = -1$	1/6	1/8	1/24	0
$x = 0$	1/12	1/12	1/12	1/12
$x = 1$	0	1/24	1/8	1/6

- (一) What is  $Cov(X,Y)$ ? (10%)
- (二) Let  $\hat{X}(Y) = aY$  be a linear estimator of  $X$ . Find  $a^*$ , the optimal value of  $a$  that minimize the mean square error. (10%)

三. Random variable  $X$  has CDF

$$F_X(x) = \begin{cases} 0 & x < -3 \\ 0.4 & -3 \leq x < 5 \\ 0.8 & 5 \leq x < 7 \\ 1 & x \geq 7 \end{cases}$$

- (一) What is the conditional CDF  $F_{X|X>0}(x)$ ? (10%)
- (二) What is the conditional PMF  $P_{X|X>0}(x)$ ? (10%)

四. Let  $K_1, K_2, \dots, K_n$  denote a sequence of independent and identical distributed Bernoulli( $p$ ) random variables.

- (一) What is the MGF  $\phi_K(s)$ ? (10%)
- (二) Let  $M = K_1 + K_2 + \dots + K_n$ , what is the MGF  $\phi_M(s)$ ? (10%)

五. Let  $X_1, X_2, \dots, X_n$  denote a sequence of independent and identical distributed sequence of exponential random variables, each with expected value 5.

- (一) Let  $M_n(X) = (X_1 + X_2 + \dots + X_n)/n$ . What is  $Var(M_9(X))$ , the variance of the sample mean based on 9 trials? (10%)
- (二) Use the central limit theorem to estimates  $P(M_9(X) > 7)$ , the probability that the sample mean of 9 trials exceeds 7. (10%)