注意事項：
1. 本試題共五題，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (20%) Find the solution of $y'' - 3y' + 2y = 2x + 8\sin(2x)$, $y(0) = \dfrac{27}{10}$,

$y'(0) = 0$.

2. (20%) Solve the following initial value problem by Laplace transform:

$$\begin{cases} \dfrac{dx}{dt} = x - 5y \\ \dfrac{dy}{dt} = -3x - 7y \end{cases} \qquad \begin{array}{l} x(0) = 2 \\ y(0) = 2 \end{array}$$

3. (20%) For each of the following statements, if you think the statement is correct, then give a proof to prove that the statement is correct; otherwise give a counterexample to show that the statement is incorrect.

   (1) (5%) $n \times n$ matrix A has n distinct eigenvalues if and only if A is diagonalizable.

   (2) (5%) Assume the matrix inverse exist, $\left(A^{-1} + B^{-1} \right)^{-1} = A\left(A + B \right)^{-1} B$

   (3) (5%) If any three vectors v_1, v_2, v_3 in R^n are linearly independent, then the vectors $w_1 = v_1 + v_2, w_2 = v_1 + v_3, w_3 = v_2 + v_3$ are also linearly independent.

   (4) (5%) If A and B are two $n \times n$ matrices, $\left(AB \right)^T = A^T B^T$.

4. (15%) Please use **Cayley-Hamilton Theorem** to find the e^A.

   where $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

5. (25%) In a certain town 30 percent of the married women get divorced each year and 20 percent of the single women get married each year. There are 8000 married women and 2000 single women and the total population remains constant.

   (1)(10%) Find the number of married women and single women after 5 years.

   (2)(15%) What will be the long-range prospects if these percentages of marriages and divorces continue indefinitely into the future?