

110EE05

國立臺北科技大學 110 學年度碩士班招生考試

系所組別：2151 電機工程系碩士班戊組

第一節 線性代數 試題 (選考)

第 1 頁 共 1 頁

注意事項：

1. 本試題共 6 題，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。
4. 所有題目均須詳細推導，否則酌予扣分。

1. (15%) Let

$$A = \begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix} \text{ and } \mathbf{u} = \begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix}$$

(1) (10%) Solve $A\mathbf{x} = \mathbf{0}$ with the reduced echelon form of the augmented matrix $[A \ \mathbf{0}]$.

(2) (5%) Determine if vector \mathbf{u} is in the null space of A .

2. (20%) Let U be an $m \times n$ column-wise orthonormal matrix and V be an $m \times m$ unitary matrix.

(1) (5%) Show that $\|VU\mathbf{x}\| = \|\mathbf{x}\|$, where \mathbf{x} is a $n \times 1$ vector.

(2) (5%) Find the determinant of $V^H V$.

(3) (5%) Find the trace of $U^H U$.

(4) (5%) Find the trace of $V^H V$.

3. (35%) Prove or disprove the following statements:

(1) (5%) If matrix A is diagonalizable and invertible, then so is A^{-1} .

(2) (5%) If a is an eigenvalue of an invertible matrix A , then a^{-1} is an eigenvalue of A^{-1} .

(3) (5%) If $A = QR$ with Q invertible, then A is similar to $B = RQ$.

(4) (5%) If A^2 is the zero matrix, then the only eigenvalue of A is zero.

(5) (5%) The set of vectors $\{\mathbf{0}, \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linear dependent set.

(6) (5%) If vectors \mathbf{v}_1 and \mathbf{v}_2 are independent, then they are orthogonal.

(7) (5%) If vectors \mathbf{v}_1 and \mathbf{v}_2 are orthogonal, then they are independent.

4. (10%) Detail the following matrix decompositions.

(1) (5%) Singular value decomposition of an $m \times n$ matrix A with rank r .

(2) (5%) QR decomposition an $m \times n$ matrix A with linearly independent columns.

5. (10%) Find a least-squares solution of $A\mathbf{x} = \mathbf{b}$ for

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$$

6. (10%) Consider the following matrix H

$$H = \begin{bmatrix} 3 & -1 & 2 & -5 \\ 0 & 5 & -3 & -6 \\ -6 & 7 & -7 & 4 \\ -5 & -8 & 0 & 9 \end{bmatrix}$$

(1) (5%) Find the determinant of A .

(2) (5%) Prove or disprove A is invertible.