

國立臺北科技大學 110 學年度碩士班招生考試

系所組別：2140 電機工程系碩士班丁組

第一節 機率 試題

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注意事項：

1. 本試題共 5 題，每題 20 分，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一. For any random variable X , show that:

- (一) $E[X - \mu] = 0$. (5%)
- (二) $E[aX + b] = aE[X] + b$. (5%)
- (三) $\text{Var}[X] = E[X^2] - \mu^2$. (5%)
- (四) $\text{Var}[aX + b] = a^2\text{Var}[X]$. (5%)

二. Assume X is an uniform (a, b) random variable, show that:(一) The CDF of X is

$$F_X(x) = \begin{cases} 0 & x \leq a \\ (x - a)/(b - a) & a < x \leq b \\ 1 & x > b \end{cases} \quad (5\%)$$

(二) The expected value of X is $E[X] = (b + a)/2$. (5%)(三) The variance value of X is $\text{Var}[X] = (b - a)^2/12$. (10%)三. Assume random variables X and Y have the following joint PDF.

$$(一) \text{ Assume } f_{X,Y}(x, y) = \begin{cases} 2 & 0 \leq y \leq 1, 0 \leq x \leq 1, x + y \leq 1 \\ 0 & \text{otherwise} \end{cases},$$

find the PDF of $X+Y$. (10%)

$$(二) \text{ Assume } f_{X,Y}(x, y) = \begin{cases} 4xy & 0 \leq y \leq 1, 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases},$$

prove that X and Y are independent. (10%)四. For a random variable X , the moment generating function (MGF) of X is defined as $\phi_X(s) = E[e^{sX}]$.(一) Prove that $E[X^n]$, the n -th moment of the random variable X , equal to

$$\left. \frac{d^n \phi_X(s)}{ds^n} \right|_{s=0}. \quad (10\%)$$

(二) Assume $\phi_X(s) = 10/(10 - s)$. Find $\text{Var}[X]$. (10%)五. In a subway station, assume the arrival time of the n -th train is $X_1 + X_2 + \dots + X_n$ where X_1, X_2, \dots, X_n are i.i.d. exponential random variables with $E[X_i] = 2$ minutes. Let $W = X_1 + X_2 + X_3$ represent the time required to wait for 3 trains.(一) Find $E[W]$ and $\text{Var}[W]$. (10%)(二) Use the central limit theorem to estimate $P(W > 20)$, the probability that waiting for more than 20 minutes. (10%)