110E02

國立臺北科技大學 110 學年度碩士班招生考試

系所組別：2131 電機工程系碩士班丙組

第一節　工程數學　試題（選考）

注意事項：

1. 本試題共六題，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (20%) Solve the differential equation $\frac{dy}{dx}-2y=-e^{3x}y^2$.

2. (15%) Solve the differential equation $x^2y''-5xy'+9y=2x^3+\ln(x), \quad x>0$.

3. (15%) Solve the equation $y''(t)+2y'(t)+2y(t)=f(t); \quad y(0)=0, y'(0)=1$

where $f(t)=\begin{cases}0 & t<1\\ 2t-3 & 1\le t<3\\ 0 & t\ge 3\end{cases}$

4. Consider the following linear system. Determine the conditions imposed on *a,* b, and *c* for each case:

(1) (5%) unique solution;

(2) (10%) many solutions;

(3) (5%) no solutions.

$$\begin{bmatrix}1 & 2 & 3\\ 4 & a & 6\\ 2 & 4 & b\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}1\\ 2\\ c\end{bmatrix}.$$

5. (20%) Compute A^{-86}, where $A=\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$.

6. (10%) Given a set $V=\left\{\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix},\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix},\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix},\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\right\}$, determine whether V forms a basis for the vector space of 2×2 real matrices.
(Remark: you have to be able to explain how you got the answer. No credit if only a simple answer is given.)