

國立臺北科技大學 110 學年度碩士班招生考試

系所組別：1501、1502、1503、1504

自動化科技研究所

第一節 工程數學 試題

第 1 頁 共 1 頁

注意事項：

1. 本試題共五題，每題 20 分，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，並詳列過程否則不予計分。

1. (20%) This question is about the LU decomposition or factorization of the matrix A , where $A = LU$.

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 4 & 1 & 4 \\ 3 & 6 & 3 & 9 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & x & 0 \\ 3 & y & z \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & u & v & w \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- (a) (10%) Find $x+y+z+u+v+w = ?$
- (b) (10%) Compute the inverse of the following matrix B or show that B is not invertible.

$$B = \begin{bmatrix} -4 & 0 & 5 \\ -3 & 3 & 5 \\ -1 & 2 & 2 \end{bmatrix}$$

2. (20%) This question is about the following matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}$$

- (a) (10%) Find the eigenvalues of $A^T A$.
- (b) (10%) Find a complete set of orthonormal eigenvectors of AA^T .

3. (20%) Answer the following questions:

- (a) (10%) Consider the line $2x + 3y = 1$. Find a vector perpendicular to the line and explain why.
- (b) (10%) Suppose A is an $m \times n$ matrix whose columns are linearly independent. What is the nullity of A ?

4. (20%) Find the determinant of the following matrices.

- (a) (10%)

$$A = \begin{bmatrix} 5 & 4 & 0 & 0 & 0 \\ 6 & 7 & 0 & 0 & 0 \\ 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 0 & 1 \end{bmatrix}$$

- (b) (10%)

$$B = \begin{bmatrix} 1 & b & b^2 \\ b & b^2 & b^3 \\ b^2 & b^3 & b^4 \end{bmatrix}, \text{ where } b \text{ is any real number}$$

5. (20%) For the following matrix

$$A = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation given by $T(\vec{x}) = A\vec{x}$.

- (a) (10%) Describe T geometrically.
- (b) (10%) Compute A^{2011} .